

Effect of interface debonding on matrix multicracking evolution of fiber-reinforced ceramic-matrix composites

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Abstract: An analytical methodology was developed to investigate the effect of fiber/matrix interface debonding on matrix multicracking evolution of fiber-reinforced CMCs (ceramic-matrix composites). The Budiansky-Hutchinson-Evans shear-lag model was adopted to analyse the micro-stress field of the damaged composites. The critical matrix strain energy criterion, which presupposes the existence of an ultimate or critical matrix strain energy with matrix, was obtained to simulate the matrix multicracking evolution of CMCs. With the increase of the applied stress, the matrix multicracking and fiber/matrix interface debonding occurred to dissipate the additional energy entered into the composites. The fiber/matrix interface debonded length under matrix multicracking evolution was obtained by treating the interface debonding as a particular crack propagation problem. The conditions for no-debonding and debonding during the evolution of matrix multicracking were discussed in terms of two interfacial properties, i. e., the interface shear stress and interface debonded toughness. When the fiber/matrix interface was bonded, the matrix multicracking evolution was much more intense compared with the interface debonding; when the fiber/matrix interface was debonded, the matrix crack density increased with the increasing of interface shear stress and interface debonded energy. The theoretical results were compared with experimental data of unidirectional SiC/CAS (calcium alumina silicate), SiC/CAS-II and SiC/borosilicate composites.

Key words: ceramic-matrix composites (CMCs); matrix cracking; interface debonding; micromechanics; critical matrix strain energy

CLC number: V21

Document code: A

Ceramic materials possess high strength and modulus at elevated temperatures. But their use as structural components is severely limited because of their brittleness. The continuous fiber-reinforced CMCs (ceramic-matrix composites), by incorporating fibers in ceramic matrices, however, not only exploit their attractive high-temperature strength, but also reduce the propensity for catastrophic failure. These materials have already been implemented on some aero-

engines' components^[1]. The CMCs flaps for exhaust nozzles of SNECMA M53 and M88 aero-engines have been used for more than one decade^[2]. The CMCs turbine vanes have been designed and tested in aero-engine environment under the implementation of UEET (ultra efficient engine technology) program^[3]. A CMCs turbine blade has been tested for 4 h by GE (General Electric) in a modified GE F414 engine, which represents the first application of CMCs material

Received: 2014-07-19

Foundation items: Supported by the Natural Science Foundation of Jiangsu Province (Grant No. BK20140813); Postdoctoral Science Foundation of China (Grant No. 2012M511274); Introduction of Talents Scientific Research Foundation of Nanjing University of Aeronautics and Astronautics (Grant No. 56YAH12034).

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in a rotating engine part. Incorporating the CMCs turbine blades on a GE 90-sized engine, the overall weight can be reduced by approximately 455 kg, which represents approximately 6% of dry weight of full sized GE 90-115^[4]. The CMCs combustion chamber floating wall tiles have also been tested in aero-engine environment for 30 min, with the temperature range of 1047—1227 °C and the pressure of 2 MPa^[5].

The fiber/matrix interfacial properties have been identified as the key factor to develop a successful fiber-reinforced CMCs from both theoretical analysis and experimental investigations at room and elevated temperatures^[6]. Considering coupling between the fiber and the matrix, the fiber/matrix interface can be classified into two cases, i. e. , the frictional interface and chemical bonded interface^[7]. For the frictionally constrained interface, several modeling methods have been adopted to predict the matrix multicracking evolution. These approaches include the maximum stress criterion, the energy balance approach and statistical failure approach. The maximum stress criterion^[8] assumes that a new matrix crack would form whenever the matrix stress exceeds the matrix strength, which is assumed to be single-valued and a known material property. The energy balance failure criteria involves the calculation of energy balance relationship before and after the formation of a single dominant cracking as originally proposed by Aveston, et al.^[9] The progression of matrix cracking determined by energy criterion is dependent upon the matrix strain energy release rate. The energy criterion is represented by Zok and Spearing^[10], Zhu and Weitsman^[11]. The statistical failure approach^[12] assumes that matrix multicracking is governed by statistical relations, which relate the size and spatial distribution of matrix flaws to their relative propagation stress. Besides the above-mentioned theoretical analysis, Morscher, et al.^[13] performed an investigation on the quasi-static loading/unloading cyclic tensile behavior in 2D SiC/SiC composite. Under tensile loading, the acoustic emission

method can be used to monitor the matrix multicracking evolution. After the specimen failure, the saturation matrix crack density in the longitudinal yarns has been measured. By combining the measured saturation matrix crack spacing and cumulated acoustic emission energy, the matrix multicracking evolution of SiC matrix could be well predicted when the applied stress increases. The stress-dependent matrix cracking was found to be dependent upon the stress in the load-bearing matrix as dictated by the architecture and constituent content. Gawayed, et al.^[14] utilized the geometric model and shear-lag theory to estimate the matrix crack density in fabric reinforced composites.

When fiber is chemically bonded to matrix, the interfacial debonding may be initialized by transverse tensile stress in front of matrix cracking tip. Solti, et al.^[7] developed an approach of critical matrix strain energy criterion to simulate the matrix multicracking evolution, in which the maximum interface shear strength criterion was adopted to determine the interface debonded length under matrix multicracking. However, following the arguments of Gao, et al.^[15], Stang and Shah^[16], the fracture mechanics approach is preferred to the shear strength approach for the interface debonding problem.

In this paper, the effect of fiber/matrix interface debonding on the matrix multicracking evolution of fiber-reinforced CMCs has been investigated. By adopting the Budiansky-Hutchinson-Evans shear-lag model, which includes the matrix shear deformation in the interface bonded region and friction in the interface debonded region, the micro-stress field of the damaged composite has been analyzed. The interface debonded length is determined by treating fiber/matrix interface debonding as a particular crack propagation problem. By adopting the critical matrix strain energy criterion, the relationship between the applied stress and matrix crack spacing can be derived. The conditions for no-debonding and debonding during the process of matrix multicracking evolution have been analyzed in terms

of two interfacial properties, i. e., the interface shear stress and interface debonded toughness. The theoretical results were compared with experimental data of unidirectional SiC/CAS, SiC/CAS-II and SiC/borosilicate composites.

1 Stress analysis

To analyze stress distributions in the fiber and matrix of the damaged composite, a unit cell is extracted from CMCs, as shown in Fig. 1. The unit cell contains a single fiber surrounded by a hollow cylinder of matrix. The fiber radius

$$\sigma_f(x) = \begin{cases} \frac{\sigma}{\varphi_f} - \frac{2\tau_i}{r_f}x, & x \in (0, L_d) \\ \sigma_{f0} + \left(\frac{\varphi_m}{\varphi_f}\sigma_{m0} - 2\frac{L_d}{r_f}\tau_i\right)\exp\left(-\rho\frac{x-L_d}{r_f}\right), & x \in (L_d, L/2) \end{cases} \quad (1)$$

$$\sigma_m(x) = \begin{cases} 2\tau_i \frac{\varphi_f}{\varphi_m} \cdot \frac{x}{r_f}, & x \in (0, L_d) \\ \sigma_{m0} - \left(\sigma_{m0} - 2\tau_i \frac{\varphi_f}{\varphi_m} \cdot \frac{L_d}{r_f}\right)\exp\left[-\frac{\rho(x-L_d)}{r_f}\right], & x \in (L_d, L/2) \end{cases} \quad (2)$$

$$\tau_i(x) = \begin{cases} \tau_i, & x \in (0, L_d) \\ \frac{\rho}{2} \cdot \left(\frac{\varphi_m}{\varphi_f}\sigma_{m0} - 2\tau_i \frac{L_d}{r_f}\right)\exp\left[-\frac{\rho(x-L_d)}{r_f}\right], & x \in (L_d, L/2) \end{cases} \quad (3)$$

where $\sigma_f(x)$, $\sigma_m(x)$ and $\tau_i(x)$ denote the fiber axial stress, matrix axial stress and fiber/matrix interface shear stress, respectively; φ_m denotes the matrix volume fraction; τ_i denotes the fiber/matrix interface shear stress; ρ denotes the Budiansky-Hutchinson-Evans shear-lag model parameter.^[17]

$$\rho^2 = \frac{4E_c G_m}{\varphi_m E_m E_f \phi} \quad (4)$$

where G_m denotes the matrix shear modulus, and

$$\phi = -\frac{2\ln\varphi_f + \varphi_m(3 - \varphi_f)}{2\varphi_m^2} \quad (5)$$

Where σ_{f0} and σ_{m0} denote the fiber and matrix axial stresses in the interface bonded region, respectively.

$$\sigma_{f0} = \frac{E_f}{E_c}\sigma + E_f(\alpha_c - \alpha_f)\Delta t \quad (6)$$

$$\sigma_{m0} = \frac{E_m}{E_c}\sigma + E_m(\alpha_c - \alpha_m)\Delta t \quad (7)$$

where E_f , E_m and E_c denote the fiber, matrix and composite elastic modulus, respectively; α_f , α_m and α_c denote the fiber, matrix and composite thermal expansion coefficient, respectively; Δt

is r_f , and the matrix radius is R ($R = r_f/\varphi_f^{1/2}$). The length of the unit cell is $L/2$, which is half of matrix crack spacing. The interface debonded length is L_d . At the matrix cracking plane, fibers carry all the loads (σ/φ_f , σ denotes the far-field applied stress, φ_f denotes the fiber volume fraction). The shear-lag model adopted by Budiansky, Hutchinson and Evans^[17] is obtained to perform stress and strain calculations in the interface debonded region ($x \in [0, L_d]$) and interface bonded region ($x \in [L_d, L/2]$).

denotes the temperature difference between fabricated temperature t_0 and room temperature t_1 ($\Delta t = t_1 - t_0$). The axial elastic modulus of the

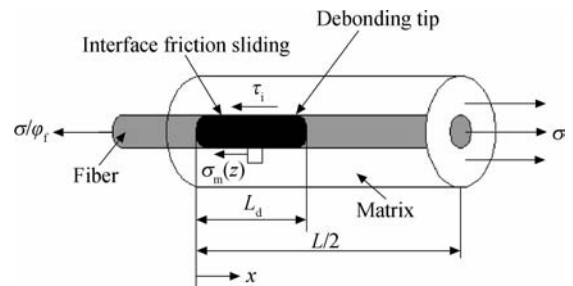


Fig. 1 Unit cell of Budiansky-Hutchinson-Evans shear-lag model

composite is approximated by rule of mixture.

$$E_c = \varphi_f E_f + \varphi_m E_m \quad (8)$$

2 Interface debonding

When matrix cracking propagates to fiber/matrix interface, it deflects along the interface. There are two approaches for the problem of fiber/matrix interface debonding, i. e., the shear strength approach and the fracture mechanics approach. The shear strength approach^[18] is

based upon a maximum shear stress criterion in which the interface debonding occurs when the interface shear stress in fiber/matrix interface approaches the shear strength of interface. On the other hand, the fracture mechanics approach^[15] treats the interface debonding as a particular crack propagation problem, i. e., the interface debonding occurs when the strain energy release rate of fiber/matrix interface achieves the debonding toughness. Sun and Singh^[19] investigated the matrix multicracking and fiber/matrix interfacial debonding by directly observing and analyzing in a transparent SiC fiber-reinforced borosilicate glass composite. For a very short fiber/matrix interface debonded length, the shear strength approach and the fracture mechanics approach gave an estimate close to experimental data. However, for a long fiber/matrix interface debonded length, the fracture mechanics approach provided a better fit. The fracture mechanics approach is adopted in the present analysis to determine the interface debonded length under the matrix multicracking evolution.^[15]

$$\zeta_d = \frac{F}{4\pi r_f} \cdot \frac{\partial w_f(0)}{\partial L_d} - \frac{1}{2} \int_0^{L_d} \tau_i \frac{\partial v(x)}{\partial L_d} dx \quad (9)$$

where $F = \pi r_f^2 \sigma / \varphi_f$, denotes the fiber load at the cracking plane, ζ_d denotes the interface debonded toughness, $w_f(0)$ denotes the fiber axial displacement at the cracking plane, $v(x)$ denotes the relative displacement between the fiber and the matrix. The axial displacements of fiber and matrix $w_f(x)$ and $w_m(x)$ are

$$w_f(x) = \int_x^{L/2} \frac{\sigma_f}{E_f} dx = \frac{\sigma}{\varphi_f E_f} (L_d - x) - \frac{\tau_i}{r_f E_f} (L_d^2 - x^2) - \frac{2\tau_i}{\rho E_f} L_d + \frac{r_f \varphi_m E_m}{\rho \varphi_f E_f E_c} \sigma + \frac{\sigma}{E_c} (L/2 - L_d) \quad (10)$$

$$w_m(x) = \int_x^{L/2} \frac{\sigma_m}{E_m} dx = \frac{\varphi_f \tau_i}{\varphi_m E_m r_f} (L_d^2 - x^2) + \frac{2\varphi_f \tau_i}{\rho \varphi_m E_m} L_d - \frac{r_f}{\rho E_c} \sigma + \frac{\sigma}{E_c} (L/2 - L_d) \quad (11)$$

Using Eqs. (10) and (11), the relative displacement between the fiber and the matrix of $v(x)$, is

$$v(x) = |w_f(x) - w_m(x)| = \frac{\sigma}{\varphi_f E_f} (L_d - x) - \frac{\tau_i E_c}{\varphi_m E_m E_f r_f} (L_d^2 - x^2) - \frac{2\tau_i E_c L_d}{\rho \varphi_m E_m E_f} + \frac{r_f}{\rho \varphi_f E_f} \sigma \quad (12)$$

Substituting $w_f(x=0)$ and $v(x)$ into Eq. (9), leads to the form of

$$\frac{E_c \tau_i^2}{\varphi_m E_m E_f r_f} L_d^2 + \left(\frac{E_c \tau_i^2}{\rho \varphi_m E_m E_f} - \frac{\tau_i \sigma}{\varphi_f E_f} \right) L_d + \frac{r_f \varphi_m E_m \sigma^2}{4 \varphi_f^2 E_f E_c} - \frac{r_f \tau_i}{2 \rho \varphi_f E_f} \sigma - \zeta_d = 0 \quad (13)$$

3 Matrix multicracking

Solti, et al.^[7] developed the critical matrix strain energy criterion to simulate the matrix multicracking evolution of fiber-reinforced CMCs. The concept of critical matrix strain energy presupposes the existence of an ultimate or critical strain energy beyond which matrix fails. Beyond the critical value of matrix strain energy, as more energy is entered into the composite when the applied stress increases, matrix can not support extra loads and continues to fail. The failure is assumed to consist of the formation of new cracks and fiber/matrix interface debonding, to make the total energy within matrix remain constant and equal to its critical value.

The matrix strain energy is determined by Eq. (14).

$$U_m = \frac{1}{2E_m} \int_{A_m} \int_0^L \sigma_m^2(x) dx dA_m \quad (14)$$

where A_m denotes the cross-section area of matrix in the unit cell. Substituting matrix axial stresses in Eq. (2) into Eq. (14), matrix strain energy considering the matrix multicracking and fiber/matrix interface partially debonding, is

$$U_m = \frac{A_m}{E_m} \left\{ \frac{4}{3} \left(\frac{\varphi_f \tau_i}{\varphi_m r_f} L_d \right)^2 L_d + \sigma_{mo}^2 (L/2 - L_d) + 2\sigma_{mo} \left(\frac{2\varphi_f \tau_i L_d}{\varphi_m r_f} - \sigma_{mo} \right) \left(-\frac{r_f}{\rho} \right) \left[\exp \left(-\rho \frac{L/2 - L_d}{r_f} \right) - 1 \right] + \left(\frac{2\varphi_f \tau_i L_d}{\varphi_m r_f} - \sigma_{mo} \right)^2 \left(-\frac{r_f}{2\rho} \right) \left[\exp \left(-2\rho \frac{L/2 - L_d}{r_f} \right) - 1 \right] \right\} \quad (15)$$

When fiber/matrix interface completely

debonds, the matrix strain energy is

$$U_m(\sigma, L, L_d = L/2) = \frac{A_m L^3}{6E_m} \left(\frac{\tau_i \varphi_i}{r_i \varphi_m} \right)^2 \quad (16)$$

By evaluating matrix strain energy at a critical stress of σ_{cr} , the critical matrix strain energy of U_{crm} can be obtained. The critical matrix strain energy is

$$U_{crm} = \frac{1}{2} k A_m L_0 \frac{\sigma_{mocr}^2}{E_m} \quad (17)$$

where $k(k \in [0, 1])$ is the critical matrix strain energy parameter, L_0 is the initial matrix crack spacing and σ_{mocr} is

$$\sigma_{mocr} = \frac{E_m}{E_c} \sigma_{cr} + E_m (\alpha_c - \alpha_m) \Delta t \quad (18)$$

where σ_{cr} is the critical stress corresponding to composite's proportional limit stress, i. e., the stress at which the stress-strain curve starts to deviate from linearity due to damage accumulation of matrix cracks. In the present analysis, the critical stress is defined to be Aveston-Cooper-Kelly matrix cracking stress^[9], which was determined using the energy balance criterion, involving calculation of energy balance relationship before and after the formation of a single dominant cracking. The Aveston-Cooper-Kelly model can be used to describe long-steady-state matrix cracking stress, corresponding to the proportional limit stress of tensile stress-strain curve^[6]. The Aveston-Cooper-Kelly matrix cracking stress is determined by Eq. (19)^[9].

$$\sigma_{cr} = \frac{1}{3} \left(\frac{6\varphi_i^2 E_f E_c^2 \tau_i \zeta_m}{r_i \varphi_m E_m^2} \right) - E_c (\alpha_c - \alpha_m) \Delta t \quad (19)$$

where ζ_m denotes the matrix fracture energy. However, as microcracks exist in the matrix when CMCs were cooled down from high fabrication temperature to room temperature, due to thermal expansion coefficient misfit between the fiber and the matrix, these microcracks are short-matrix-cracking, and the cracking stresses of these microcracks lie in the linear region of tensile stress-strain curve. When the applied stress increases, microcracks can propagate into long-matrix-cracking. The matrix cracking stress of Aveston-Cooper-Kelly model was used

to determine the critical matrix strain energy.

The energy relationship to evaluate the matrix multicroacking evolution is

$$U_m(\sigma > \sigma_{cr}, L, L_d) = U_{crm}(\sigma_{cr}, L_0) \quad (20)$$

The matrix multicroacking evolution versus the applied stress can be solved by Eq. (20) when the critical matrix cracking stress of σ_{cr} and the fiber/matrix interface debonded length of L_d are determined by Eqs. (19) and (13) for different interface debonding cases.

4 Discussions

Two classes of matrix multicroacking evolution problem corresponding to “frictionless” and “frictional” interface are discussed.

4.1 Frictionless interface ($\tau_i = 0$ MPa)

When fiber/matrix interface frictional shear stress is $\tau_i = 0$ MPa, the interface can be classified into two cases, i. e., the interface completely bonding and completely debonding. When fiber/matrix interface completely debonds, the matrix crack density would not change with the increase of applied stress. When fiber/matrix interface bonded during the process of matrix multicroacking evolution, the instantaneous matrix strain energy is

$$U_m = \frac{A_m}{E_m} \left\{ \sigma_{mo}^2 \frac{L}{2} + 2\sigma_{mo}^2 \left(\frac{r_f}{\rho} \right) \left[\exp\left(-\rho \frac{L}{2r_f}\right) - 1 \right] - \frac{1}{2} \sigma_{mo}^2 \left(\frac{r_f}{\rho} \right) \left[\exp\left(-\rho \frac{L}{r_f}\right) - 1 \right] \right\} \quad (21)$$

The critical matrix strain energy U_{crm} is

$$U_{crm} = \frac{1}{2} k L_0 E_m \left(\frac{\rho \varphi_i E_f}{r_f E_m} \zeta_m \right)^2 \quad (22)$$

The predicted matrix crack density versus the normalized stress of σ/σ_{cr} for the present analysis and analysis of Solti, et al.^[7] are shown in Fig. 2, in which the predicted matrix crack density of the present analysis is higher than that of Solti, et al.^[7], at the same normalized stress of σ/σ_{cr} . As the Budiansky-Hutchinson-Evans shear-lag model is adopted to describe the matrix axial stress distribution in the interface bonded

region, considering the effect of matrix shear deformation, the instantaneous matrix strain energy and matrix crack density at the same normalized stress of σ/σ_{cr} would increase.

The matrix crack density versus the normalized stress of σ/σ_{cr} for the present analysis at different fiber volume fractions of $\varphi_f=0.3$ and 0.4 are shown in Fig. 3, in which the matrix crack density increases with the increase of fiber volume fraction at the same normalized stress. When the fiber volume fraction increases, the amount of loads transferred between the fiber and the matrix would increase. The loads carried by matrix and the matrix strain energy would be increased, as shown in Eq. (16). The matrix crack spacing needed for the instantaneous matrix strain energy to approach the critical value, would be decreased. The matrix crack density increases with the increase of fiber volume fraction.

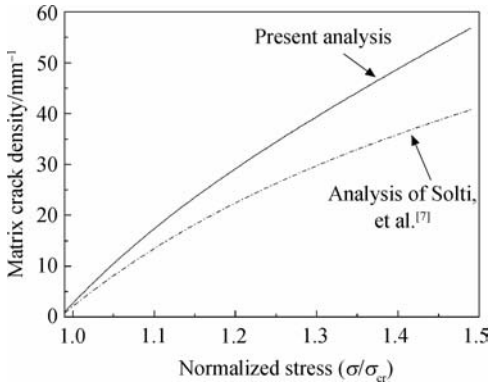


Fig. 2 Matrix crack density versus the normalized stress of σ/σ_{cr} for the present analysis and analysis of Solti, et al. [7]

4.2 Frictional interface ($\tau_i > 0$ MPa)

The fiber/matrix interface shear stress in the interface debonded region may be formed by thermal residual radial compressive stress for misfit between fiber and matrix thermal expansion coefficient. Besides, the interface shear stress may be induced by interface roughness during the relative slip between the fiber and the matrix. During the process of matrix multicracking evolution, the interface shear stress transfers loads between the fiber and the matrix. The relative slip in the interface debonded region dis-

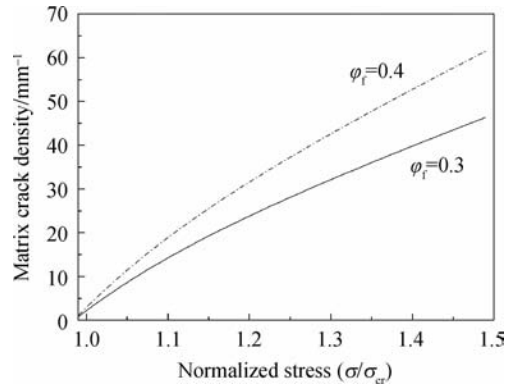


Fig. 3 Matrix crack density versus the normalized stress of σ/σ_{cr} for the present analysis with different fiber volume fraction

spitates the additional energy and affects the matrix multicracking evolution.

Two interface cases, i. e., the frictional interface without chemically bonding ($\tau_i > 0$ MPa and $\zeta_d = 0$ J/m²) and frictional interface with chemically bonding ($\tau_i > 0$ MPa and $\zeta_d > 0$ J/m²), are discussed.

4.2.1 Frictional interface without chemically bonding ($\tau_i > 0$ MPa and $\zeta_d = 0$ J/m²)

When fiber/matrix interface is no chemically bonding ($\zeta_d = 0$ J/m²), the interface debonding is only controlled by frictional interface shear stress, which transfers loads between the fiber and the matrix in the interface debonded region. The interface debonded length can be determined by Eq. (13) by inserting $\zeta_d = 0$ J/m².

$$L_d = \frac{r_f}{2} \left(\frac{\varphi_m E_m \sigma}{\varphi_f E_c \tau_i} - \frac{2}{\rho} \right) \quad (23)$$

When fiber/matrix interface debonded length is $L_d = 0$ mm, the initial interface debonded stress of σ_d , is

$$\sigma_d = \frac{2\varphi_f E_c \tau_i}{\rho \varphi_m E_m} \quad (24)$$

The critical matrix strain energy is

$$U_{crm} = \frac{1}{2} k L_0 \frac{E_m}{E_c} \left(\frac{6\varphi_f^2 E_f E_c^2 \tau_i \zeta_m}{r_f \varphi_m E_m^2} \right)^{2/3} \quad (25)$$

The interface debonded length versus the normalized stress of σ/σ_{cr} for different interface shear stress is illustrated in Fig. 4, in which the interface debonded length decreases with the increase of interface shear stress at the same normalized stress of σ/σ_{cr} . The matrix crack density

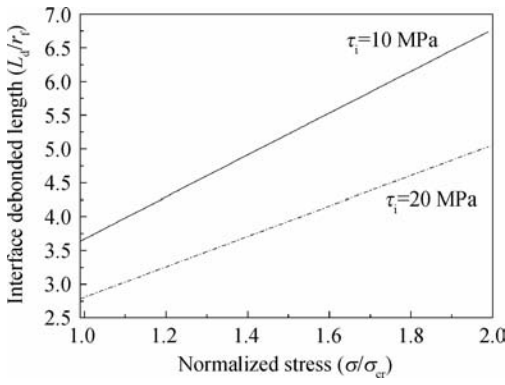


Fig. 4 Interface debonded length versus the normalized stress of σ/σ_{cr} of the present analysis with different fiber/matrix interface shear stress

versus the normalized stress of σ/σ_{cr} for different interface shear stress is shown in Fig. 5, in which the matrix crack density increases with the increase of interface shear stress at the same normalized stress of σ/σ_{cr} . When the interface shear stress increases, the loads transferred between the fiber and the matrix increase, the loads carried by matrix increase, and the interface debonded length decreases. The matrix crack spacing required for the instantaneous matrix strain energy to approach critical value, would be decreased. With the increase of applied stress, the energy dissipated through frictional slip between the fiber and the matrix decreases due to less of interface debonded length, the additional energy entered into the composite would be dissipated through forming more matrix mul-

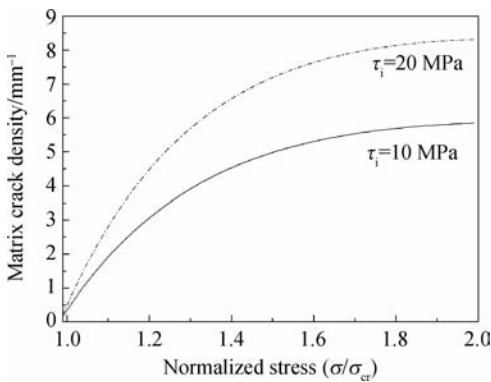


Fig. 5 Matrix crack density versus the normalized stress of σ/σ_{cr} of the present analysis with different fiber/matrix interface shear stress

ticracking. The matrix crack density increases with the increase of interface shear stress.

4.2.2 Frictional interface with chemically bonding ($\tau_i > 0$ MPa and $\zeta_d > 0$ J/m²)

When fiber/matrix interface is chemically bonded ($\zeta_d > 0$ J/m²), the interface debonding is controlled by frictional shear stress and debonded energy of fiber/matrix interface. The interface debonded length is determined by solving Eq. (13).

$$L_d = \frac{r_f}{2} \left(\frac{\varphi_m E_m \sigma}{\varphi_f E_c \tau_i} - \frac{1}{\rho} \right) - \sqrt{\left(\frac{r_f}{2\rho} \right)^2 + \frac{r_f \varphi_m E_m E_f}{E_c \tau_i^2} \zeta_d} \quad (26)$$

The initial interface debonded stress is determined by setting $L_d = 0$ mm in Eq. (26).

$$\sigma_d = \frac{\varphi_f E_c \tau_i}{\rho \varphi_m E_m} \left[1 + \sqrt{1 + \frac{4\rho^2 \varphi_m E_m E_f}{r_f E_c} \left(\frac{\zeta_d}{\tau_i^2} \right)} \right] \quad (27)$$

Budiansky, et al.^[17] investigated the interface debonding upon initial matrix cracking, and found that fiber/matrix interface debonding occurs when $\zeta_d/\zeta_m < 0.2$. The interface debonded length versus the normalized stress of σ/σ_{cr} for different interface debonded energy is illustrated in Fig. 6, in which the interface debonded length decreases with the increase of interface debonded energy. The matrix crack density versus the normalized stress of σ/σ_{cr} for different interface debonded energy is illustrated in Fig. 7, in which matrix crack density increases with the increase

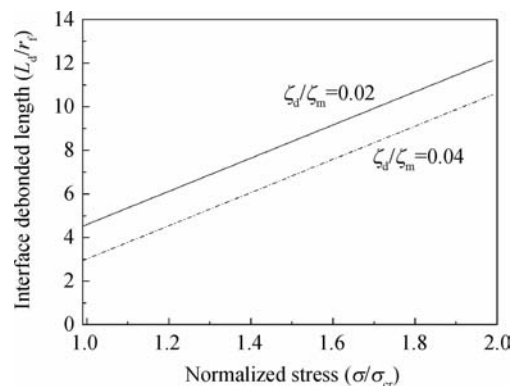


Fig. 6 Interface debonded length versus the normalized stress of σ/σ_{cr} of the present analysis with different fiber/matrix interface debonded energy

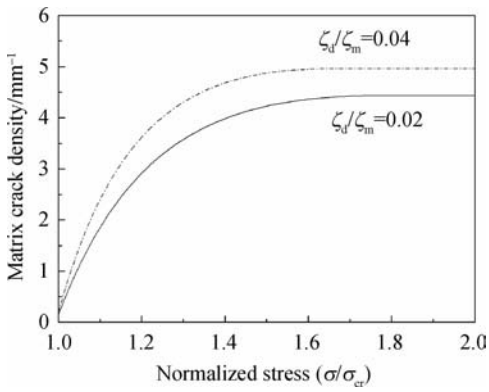


Fig. 7 Matrix crack density versus the normalized stress of σ/σ_{cr} of the present analysis with different fiber/matrix interface debonded energy

of interface debonded energy at the same normalized stress of σ/σ_{cr} . When the interface debonded energy increases, the interface debonded length decreases. With the increase of applied stress, the energy dissipated through frictional slip between the fiber and the matrix decreases due to less of interface debonded length, the additional energy entered into the composite would be dissipated through forming more matrix multicracking. The matrix crack density increases with the increase of interface debonded energy.

5 Experimental comparisons

Pryce and Smith^[20] investigated the tensile behavior of unidirectional SiC/CAS composite. Under tensile loading, the specimen was loaded to progressively higher stress levels to directly observe the matrix multicracking evolution by optical and scanning electron microscopy. The matrix crack density was determined by counting the number of cracks in a gauge-length of approximately 15 mm. The experimental matrix crack density versus the applied stress of unidirectional SiC/CAS composite is shown in Fig. 8. The initial matrix cracking stress is approximately 100 MPa, and the matrix cracks approach saturation at approximately 270 – 280 MPa. In the present analysis, the critical matrix cracking stress is determined by Aveston-Cooper-Kelly model for long-steady-state matrix cracking, and the matrix multicracking evolution of unidirec-

tional SiC/CAS composite is determined by critical matrix strain energy criterion of Eq. (20). The material properties of unidirectional SiC/CAS composite are given in Table 1. By measuring the initial matrix cracking stress and saturation matrix crack spacing, the interface shear stress is estimated by $\tau_i=10\text{MPa}$ ^[20]. Domergue, et al.^[21] estimated the interface debonded energy of unidirectional SiC/CAS composite by measuring the loading/unloading transition stress, $\zeta_d=(0.5\pm0.3)\text{J/m}^2$. In the present analysis, the interface debonded energy is given by $\zeta_d=0.4\text{J/m}^2$. The critical matrix cracking stress is given by 160 MPa. The predicted matrix multicracking evolution of unidirectional SiC/CAS composite using the present analysis and Solti, et al.^[7] analysis are illustrated in Fig. 8. When the applied stress increases, the matrix crack density increases. For the present analysis, the matrix cracks approach saturation at the applied stress of 285 MPa, the saturation matrix crack spacing is about $142\mu\text{m}$. For the analysis of Solti, et al.^[7], matrix cracks approach saturation at the applied stress of 312 MPa, the saturation matrix crack spacing is about $160\mu\text{m}$. The evolution of matrix multicracking predicted using the present analysis, including the saturation matrix crack spacing and stress, agreed better with experimental data than that of analysis of Solti, et al.^[7]

Beyerle, et al.^[22] investigated the tensile behavior of unidirectional SiC/CAS- II compos-

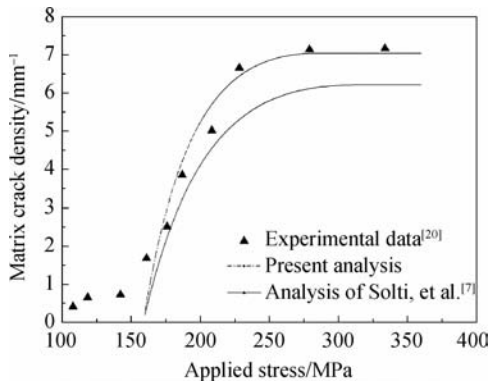


Fig. 8 Experimental and predicted matrix crack density versus the applied stress of unidirectional SiC/CAS composite

ite. Under tensile loading, the onset and evolution of matrix cracking were investigated by conducting experiments on specimens with polished surfaces in situ with an optical microscope. The acoustic emission was also used to monitor the matrix multiracking evolution. The experimental matrix crack density versus the applied stress of unidirectional SiC/CAS- II composite is illustrated in Fig. 9. The initial matrix cracking stress is approximately 120 MPa. However, these cracks correspond to microcracks, which have no effect on the nonlinear of unidirectional SiC/CAS- II composite. When the applied stress increases, these microcracks propagate to form long-steady-state matrix cracks. The matrix multiracking evolution of unidirectional SiC/CAS composite is determined by critical matrix strain energy criterion of Eq. (20). The material properties of unidirectional SiC/CAS- II composite are given in Table 1. By measuring the initial matrix cracking stress and saturation matrix crack spacing, the interface shear stress is estimated by $\tau_i = 20 \text{ MPa}^{[22]}$. Domergue, et al. ^[21] estimated the interface debonded energy of unidirectional SiC/CAS composite by measuring the loading/unloading transition stress, $\zeta_d = (0.5 \pm 0.3) \text{ J/m}^2$. In the present analysis, the interface debonded energy is given by $\zeta_d = 0.4 \text{ J/m}^2$. The critical matrix cracking stress is given by 164 MPa. The predicted matrix multiracking evolution of unidirectional SiC/CAS- II composite using the present analysis and analysis of Solti, et al. ^[7] are illustrated in Fig. 9. When the applied stress increases, the matrix crack density increases. For the present analysis, the matrix cracks approach saturation at the applied stress of 260 MPa, the saturation matrix crack spacing is about $102 \mu\text{m}$; for the analysis of Solti, et al. ^[7], the matrix cracks approach saturation at the applied stress of 335 MPa, the saturation matrix crack spacing is about $150 \mu\text{m}$. The evolution of matrix multiracking predicted using the present analysis, including the saturation matrix crack spacing and stress, agreed better with experimental data than that of analysis of Solti,

et al. ^[7]

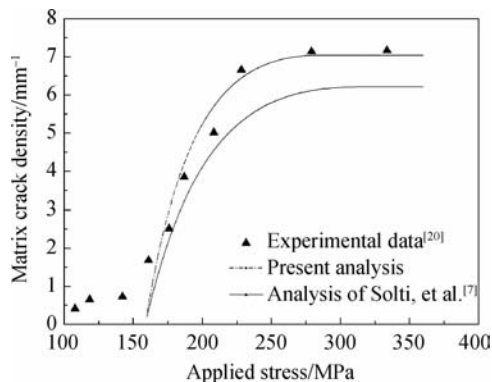


Fig. 9 Experimental and predicted matrix crack density versus the applied stress of unidirectional SiC/CAS- II composite

Okabe, et al. ^[23] investigated the tensile behavior of unidirectional SiC/borosilicate composite. The matrix crack spacing was measured under tensile loading by periodically stopping test to replicate the damage process on specimen surface. The experimental matrix crack density versus the applied stress is shown in Fig. 10, in which the initial matrix cracking stress is approximately 150 MPa, and approaches saturation at approximately 390–400 MPa. In the present analysis, the matrix multiracking evolution of unidirectional SiC/borosilicate composite is determined by critical matrix strain energy criterion of Eq. (20). The material properties of unidirectional SiC/borosilicate composite are given in Table 1. By measuring the saturation matrix crack spacing, the interface shear stress is estimated by $\tau_i = 7.6 \text{ MPa}^{[23]}$. Domergue, et al. ^[21] estimated the interface debonded energy of unidirectional SiC/CAS composite by measuring the loading/unloading transition stress, $\zeta_d = (0.5 \pm 0.3) \text{ J/m}^2$. In the present analysis, the interface debonded energy is given by $\zeta_d = 0.2 \text{ J/m}^2$. The critical matrix cracking stress is given by 220 MPa. The predicted matrix multiracking evolution of unidirectional SiC/borosilicate composite using the present analysis and analysis of Solti, et al. ^[7] are illustrated in Fig. 10. When the applied stress increases, the matrix crack density increases. For the present analysis, the matrix

cracks approach saturation at the applied stress of 340 MPa, the saturation matrix crack spacing is about 150 μm . For the analysis of Solti, et al.^[7], the matrix cracks approach saturation at the applied stress of 365 MPa, the saturation matrix crack spacing is about 170 μm . The evo-

lution of matrix multicracking predicted using the present analysis, including the saturation matrix crack spacing and stress, agreed better with experimental data than that of analysis of Solti,et al.^[7]

Table 1 Material properties of unidirectional SiC/CAS, SiC/CAS- II and SiC/borosilicate composites

Items	SiC/CAS ^[20]	SiC/CAS- II ^[22]	SiC/Borosilicate ^[23]
E_f/GPa	190	200	230
E_m/GPa	90	97	60
φ_f	0.34	0.37	0.31
$r_f/\mu\text{m}$	7.5	7.5	8
Fiber Possion ratio ν_f	0.2	0.2	0.2
Matrix Possion ratio ν_m	0.2	0.2	0.2
$\alpha_f(10^{-6}/^{\circ}\text{C})$	3.3	4	3.1
$\alpha_m(10^{-6}/^{\circ}\text{C})$	4.6	5	3.25
$\Delta t/^{\circ}\text{C}$	-1000	-1000	-1000
τ_i/MPa	10	20	7.6
$\zeta_d/(\text{J}/\text{m}^2)$	0.4	0.4	0.2
$\zeta_m/(\text{J}/\text{m}^2)$	25 ^[22]	25 ^[22]	29 ^[22]

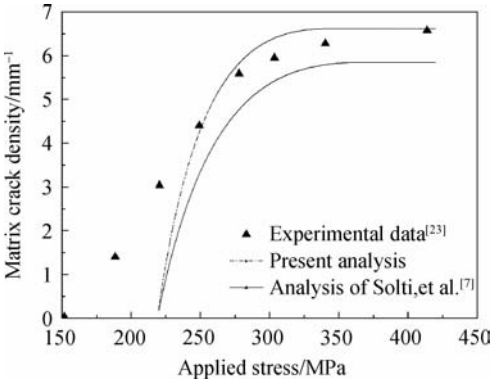


Fig. 10 Experimental and predicted matrix crack density versus the applied stress of unidirectional SiC/borosilicate composite

In the present analysis, the fracture mechanics interface debonding criterion is used to determine the interface debonded length under matrix multicracking evolution. However, in the analysis of Solti, et al.^[7], the maximum interface shear strength criterion was used to determine fiber/matrix interface debonded length,

which is only appropriate for short interface debonded length. Besides, the Budiansky-Hutchinson-Evans shear-lag model considering matrix shear deformation was used to analyze the micro-stress field of the damaged composite. The predicted results using the present analysis agreed better with experimental data than that of analysis of Solti,et al.^[7]

6 Conclusions

The effect of fiber/matrix interface debonding on matrix multicracking evolution of fiber-reinforced CMCs has been investigated. The Budiansky-Hutchinson-Evans shear-lag model, considering the effect of matrix shear deformation, is used to analyze the micro-stress field in the matrix and fibers of the damaged composite. The critical matrix strain energy criterion is used to simulate matrix multicracking evolution upon tensile loading. During the process of matrix multicracking evolution, the fracture mechanics

interface debonding criterion is used to determine fiber/matrix interface debonded length. The matrix multiracking evolution of three interface cases, i. e., the bonded interface, frictional interface without chemically bonding and frictional interface with chemical bonding, have been investigated.

1) When fiber/matrix interface is bonded, the matrix multiracking evolution is more intense compared with that of interface debonding. With the increase of fiber volume fraction, the loads transferred between the fiber and the matrix increases, the matrix crack spacing required for the instantaneous matrix strain energy to approach critical value, would be decreased. The matrix crack density increases with the increase of fiber volume fraction at the same applied stress.

2) When fiber/matrix interface is frictional bonded without chemically bonding, the loads transferred between the fiber and the matrix depend upon the interface frictional shear stress. With the increase of interfacial shear stress, the interface debonded length decreases. With the increase of applied stress, the energy dissipated through frictional slip between the fiber and the matrix decreases due to less of interface debonded length, the additional energy entered into the composite would be dissipated through forming more matrix multiracking. The matrix crack density increases with the increase of interface shear stress at the same applied stress.

3) When fiber/matrix interface is frictional bonded with chemically bonding, the interface debonded length during matrix multiracking evolution is controlled by interface shear stress and interface debonded energy. With the increase of interface debonded energy, the interface debonded length decreases. With the increase of applied stress, the energy dissipated through frictional slip between the fiber and the matrix decreases due to less of interface debonded length, the additional energy entered into the composite would be dissipated through forming more matrix multiracking. The matrix crack

density would be increased with the increase of interface debonded energy at the same applied stress.

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